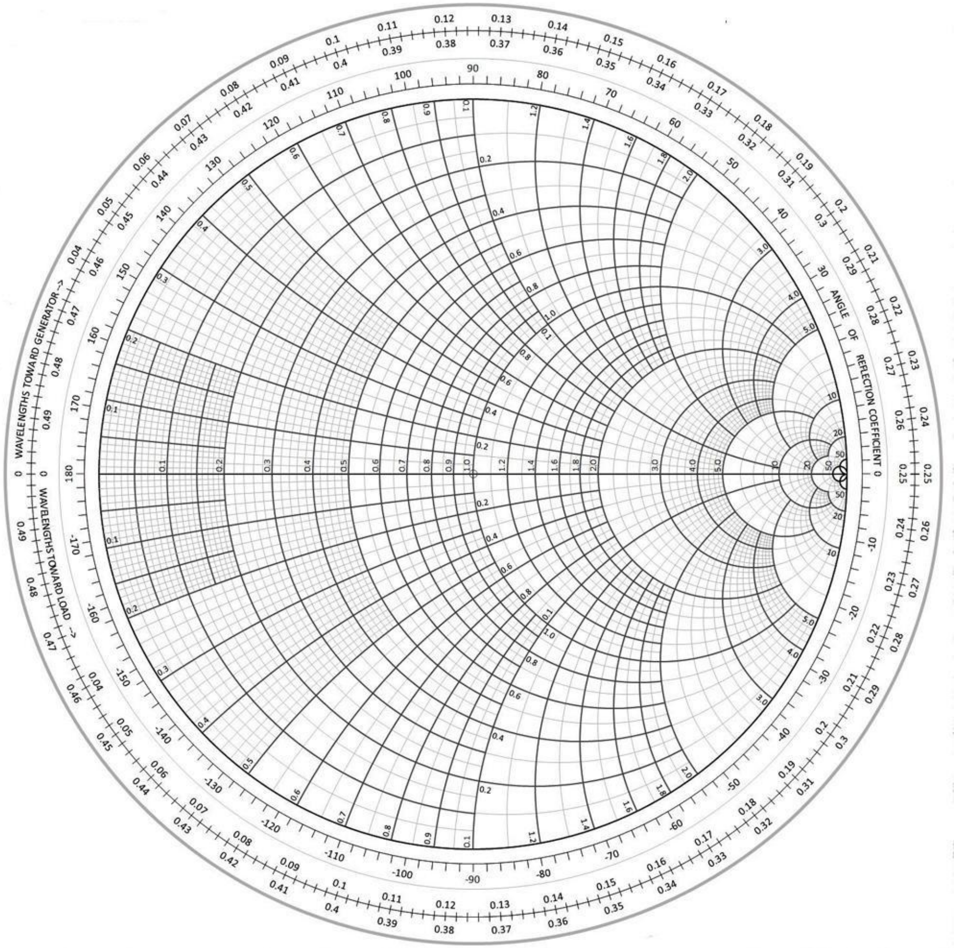
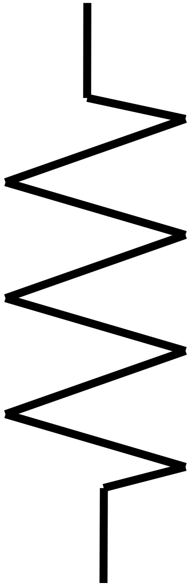
Understanding Complex Impedances using the Smith Chart

Complex impedances can be difficult to understand.

Named after its inventor, Philip Hagar Smith (1905-1987), the Smith Chart is a useful way to visualise and analyse a complex impedance.



Resistance

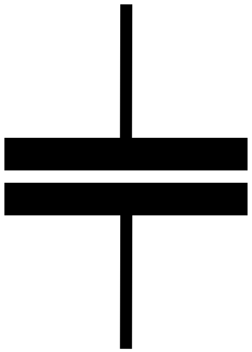


Resistance (Ω) is the property of a conductor to impede current flow.

Voltage and current are in phase.

Power is dissipated as heat.

Capacitive Reactance



Capacitive Reactance (X_c) is the property of a capacitor to impede current flow.

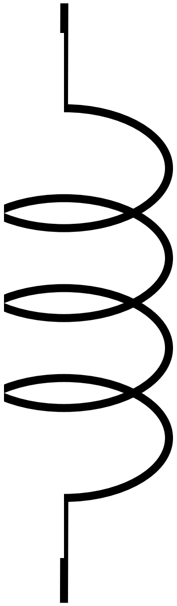
It varies with frequency. With a DC voltage applied there is no current flow at all. If an AC voltage is applied, then the reactance is dependant upon the frequency.

$$X_c = \frac{1}{2\pi f C} \text{ Ohms}$$

Current leads the voltage by 90º

Imagine this by the analogy of a battery - current needs to flow into the battery before the voltage builds up.

Inductive Reactance



Inductive Reactance (X_L) is the property of an inductor to impede current flow.

It increases with frequency. With a DC voltage applied, it appears as a short circuit. If an AC voltage is applied, then the reactance is dependant upon the frequency.

$X_L = 2\pi f L$ ohms

Voltage leads current by 90º

Real and Imaginary Quantities

While both resistance and reactance are measured in Ohms, there is a difference between the two. Resistance can turn power into heat, but reactance can only reflect power back to the source.

To differentiate between the two, we say that resistance and reactance are at right angles to each other (its a bit like saying that width and height are measured in the same units but they are completely different measurements).

Resistance is measured in ohms (Ω).

Capacitive reactance is measured in – jΩ. –j means it’s at right angles to resistance, and capacitive.

Inductive reactance is measured in + jΩ. +j means it’s at right angles to resistance, and inductive.

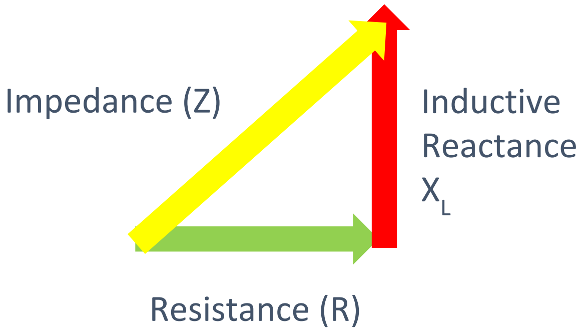
Real and Imaginary Quantities

Resistance (R)

Inductive Reactance X_L

Capacitive Reactance X_C

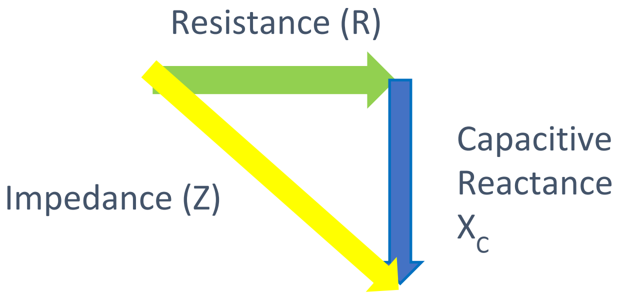
In any circuit that comprises resistance and reactance we cannot simply add the values to each other, as they are at right angles. We need to add their vectors together using Pythagoras' laws. The result is called **IMPEDANCE (Z).**



Impedance (Z)

Inductive Reactance X_L

Resistance (R)



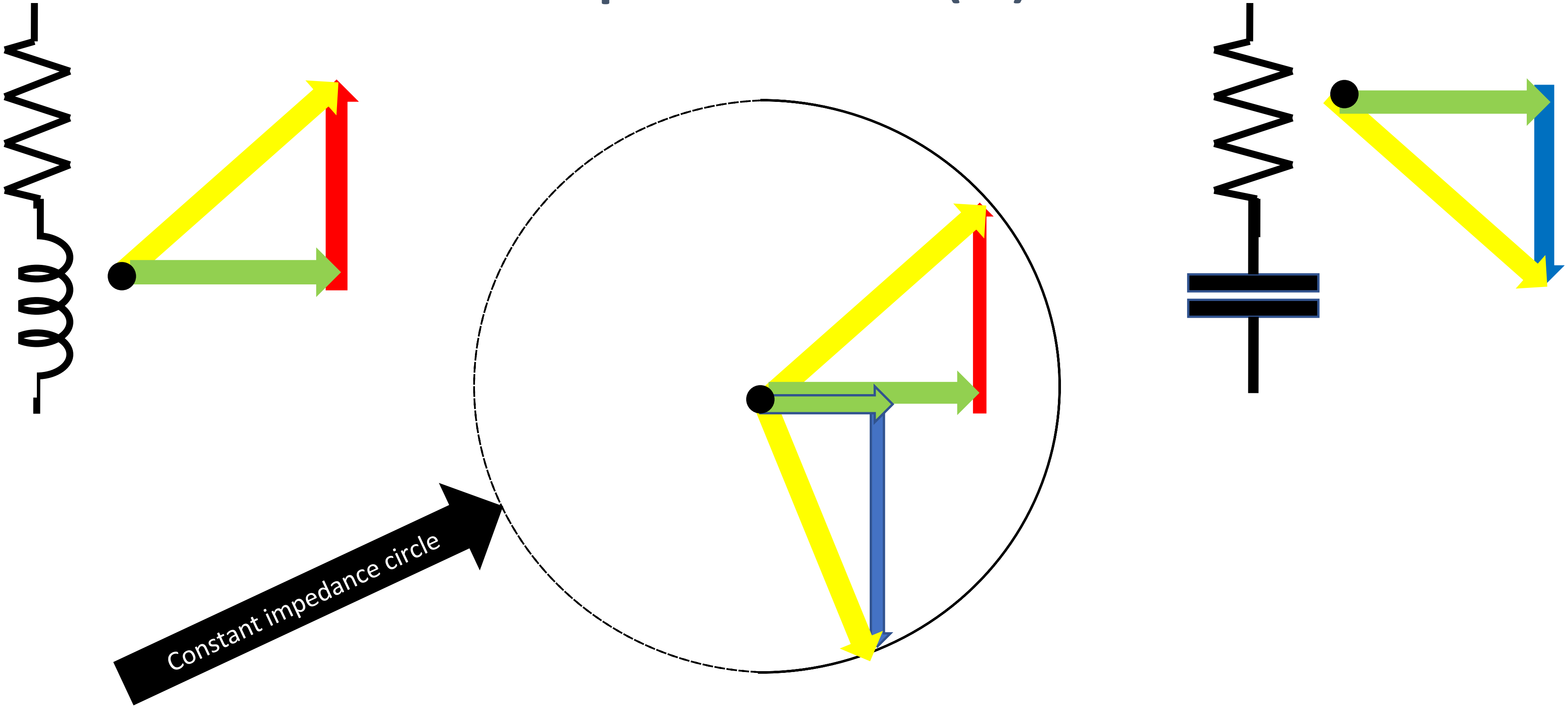
Resistance (R)

Impedance (Z)

Capacitive Reactance X_C

$$Z = \sqrt{R^2 + X_L^2} \quad \text{or} \quad Z = \sqrt{R^2 + X_C^2}$$

Impedance (Z)



The problem with defining impedance in ohms, is that it says nothing about the ratio of resistance and reactance. There are an infinite number of ways to make up any given value of impedance from resistive and reactive components.

Impedance (Z)

So we can define impedance in two ways.

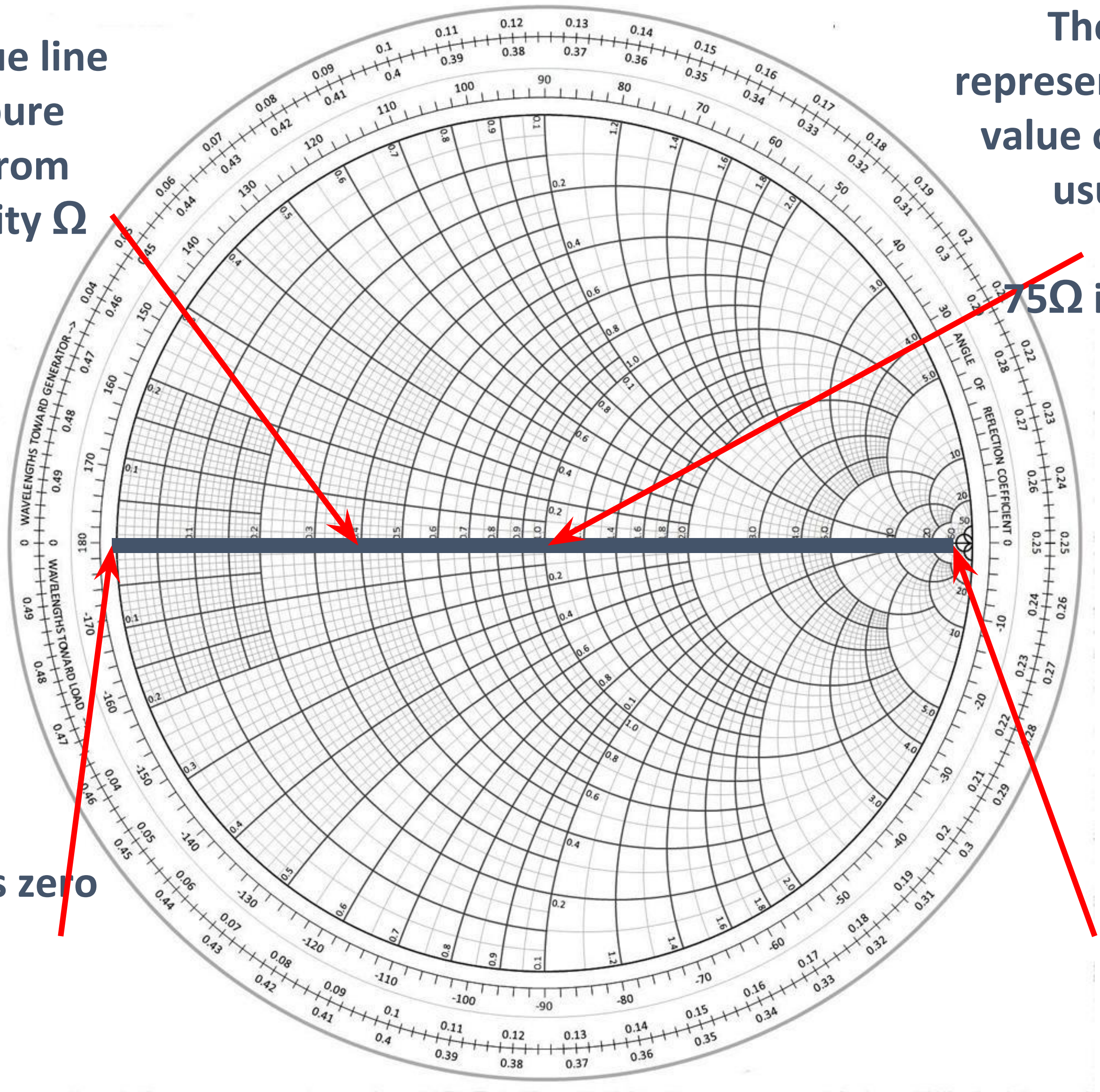
(1) As the vector addition of the real and imaginary components:

- Advantage – easy to understand.
- Disadvantage – we have lost any knowledge of the reactive components.

(2) By stating the resistive and reactive components separately. This retains details of the reactive components. For example:

$$R + jX_L \quad \text{or} \quad R - jX_C$$

It is this latter method that is displayed graphically in a Smith Chart.

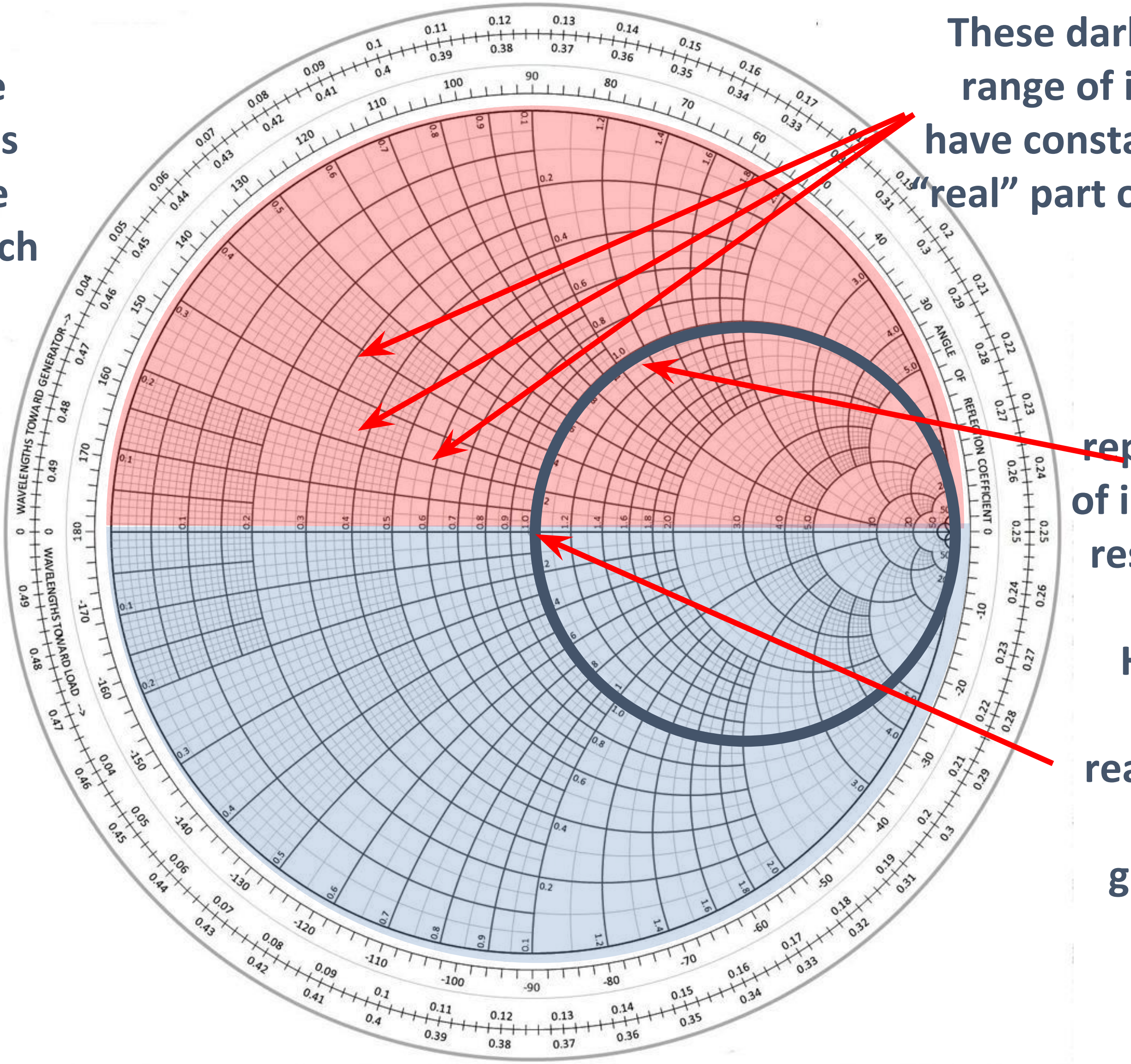


This bold blue line represents pure resistance, from zero to infinity Ω

The centre of the line represents the normalised value of the chart. This is usually 50Ω for radio applications but 75Ω is sometimes seen, and any value is possible.

This point is zero Ω, or short circuit.

This point is infinity Ω, or open circuit.



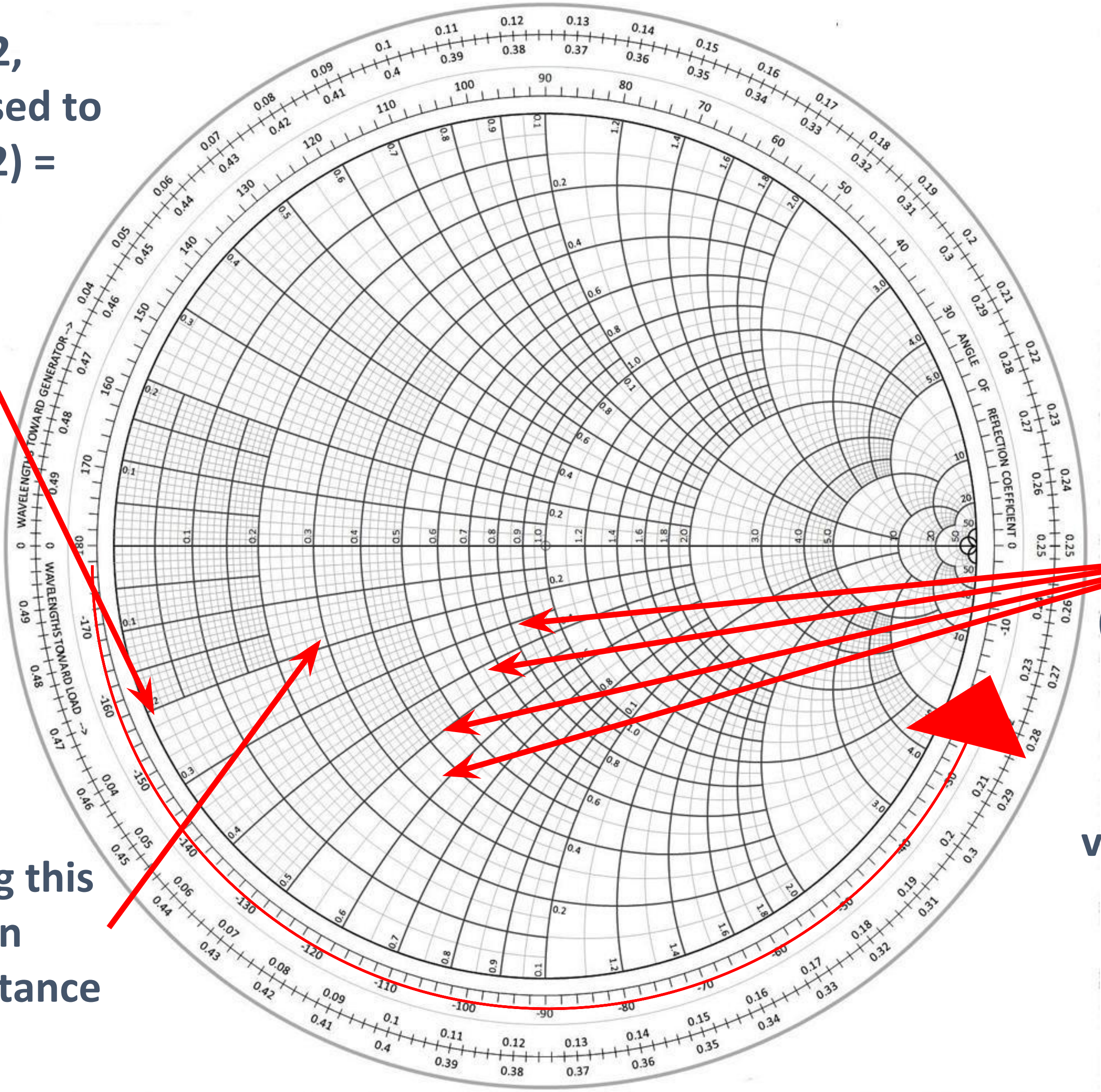
The red area represents the range of values with a positive value of j, which is inductively reactive.

The blue area represents the range of values with a negative value of j, which is capacitively reactive.

These dark lines represent a range of impedances which have constant resistance (the "real" part of the impedance).

The blue circle represents the range of impedances with a resistive component equal to 50Ω. However, only this point here has no reactive component, and is therefore a good match to 50Ω coaxial cable.

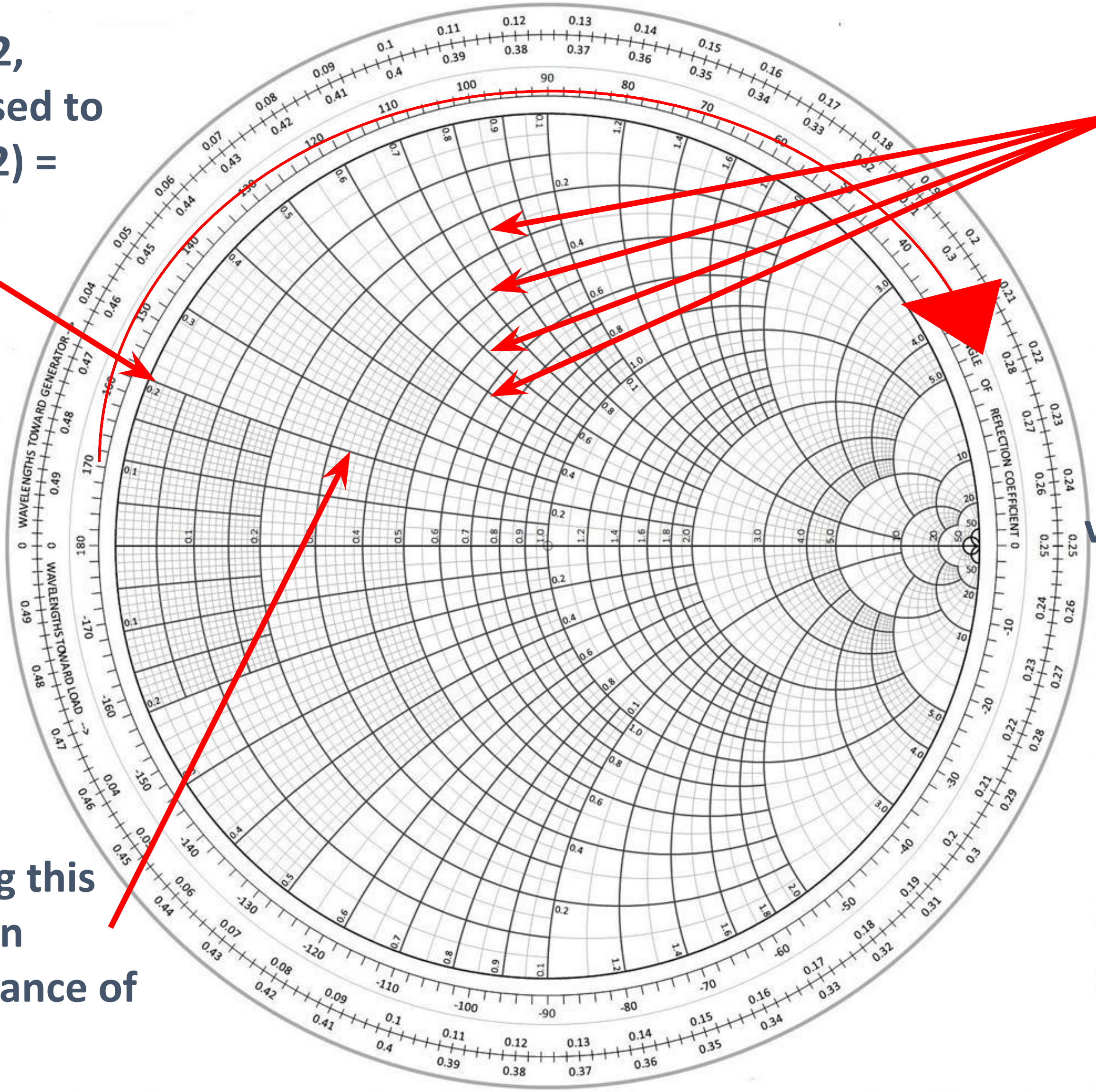
This point is 0.2, which normalised to 50Ω is (50 * 0.2) = -j10Ω



Any point along this line will have an capacitive reactance of -j10Ω

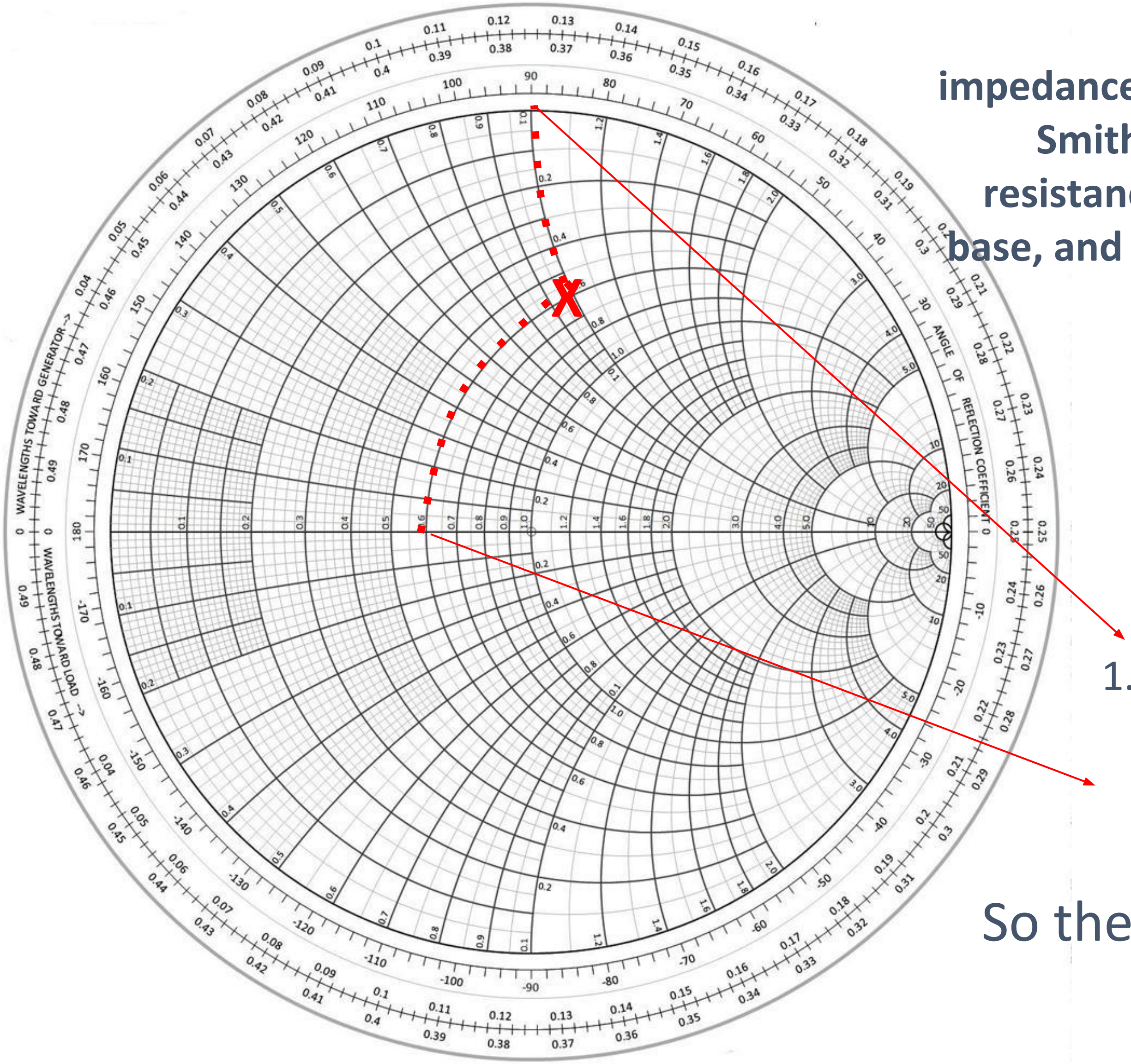
These dark lines represent a range of impedances which have constant capacitive reactance (the "imaginary" part of the impedance). The scale around the outside gives the value of the reactance relative to the normalised impedance.

This point is 0.2, which normalised to 50Ω is (50 * 0.2) = +j10Ω



These dark lines represent a range of impedances which have constant inductive reactance (the "imaginary" part of the impedance). The scale around the outside gives the value of the reactance relative to the normalised impedance.

Any point along this line will have an inductive reactance of +j10Ω

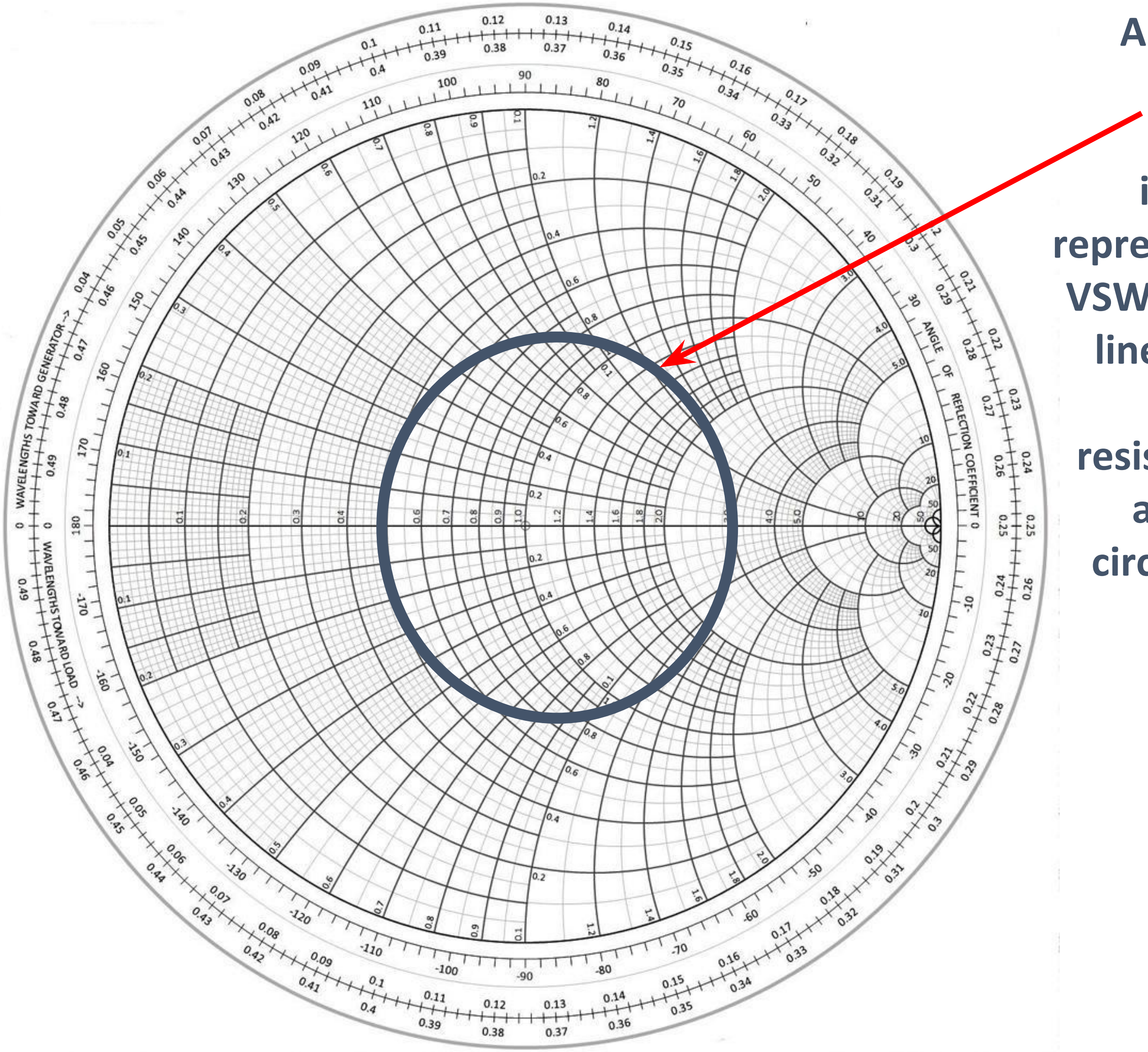


To determine the impedance of any point on a Smith Chart, follow the resistance line back to the base, and the reactance line to the outside.

1.0 x +j50Ω = **+j50Ω**

0.6 x +50Ω = **30Ω**

So the impedance is **30+j50Ω**



Any circle centred around the normalised impedance point represents a constant VSWR. The bold blue line passes through 25Ω and 100Ω resistance points, so any point on that circle is 2.0:1 VSWR